Data-Driven Decentralized Stabilization of Interconnected Systems based on Dissipativity

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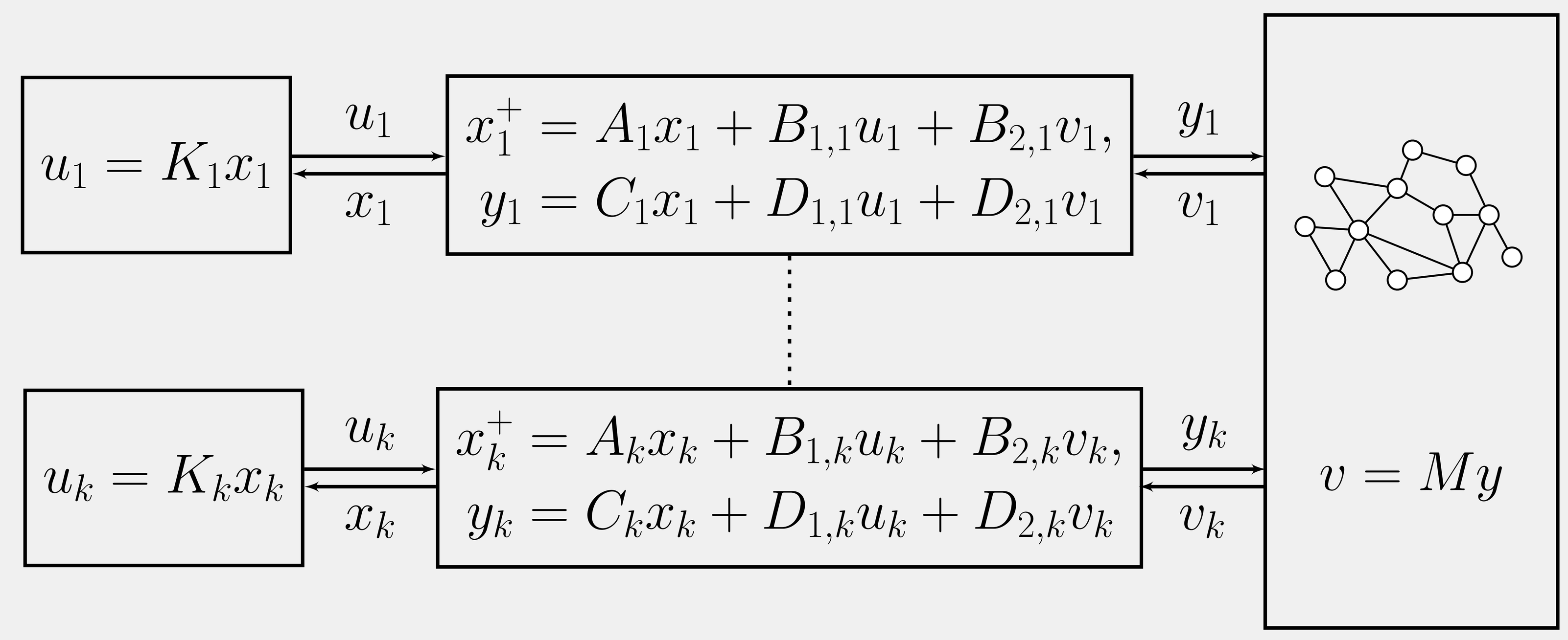
Summary

Large-scale interconnected systems: rich theory & many applications

Question: can we control interconnected systems with unknown dynamics & interconnection structure in a scalable fashion?

→ **data-driven decentralized control** based on **dissipativity & LMIs**

Problem formulation



- (A_i, B_i, C_i, D_i) & M: unknown
- Local data: $\{X_i, X_i^+, U_i, V_i, Y_i\}$ + noise in a QMI set w.r.t. Φ_i
- Interconnection data: $\{\tilde{V}_i, \tilde{Y}\}$ + noise in a QMI set w.r.t. Ψ_i
- Objective: design $\{K_i\}_{i=1}^k$ s.t. closed-loop system is asymp. stable

Dissipativity

System: dissipative w.r.t. supply rate $s(v, y) \iff \exists$ positive def. V s.t.

$$V(x^+) - V(x) \leq s(v, y),\ \forall x, v.$$

- Describes how a system stores and exchanges energy over time
- Generalization of passivity, finite-gain $\mathcal{L}_2$-stability, etc.
- We say "dissipative w.r.t. (F, G, H)" when $s(v,y) = \begin{bmatrix} v \\ y\end{bmatrix}^\top \begin{bmatrix} H & G^\top \\ G & F\end{bmatrix}^{-1} \begin{bmatrix} v \\ y\end{bmatrix}$

Data-driven decentralized LMIs

Theorem 1 (Local control design) $\exists K_i$ that $u_i = K_i x_i$ makes the i-th system dissipative w.r.t. (F_i, G_i, H_i), iff $\exists P_i > 0,\ L_i, \alpha_i \geq 0$ s.t.

$$\begin{bmatrix} P_i & 0 & 0 & 0 & 0 & 0 \\ 0 & -F_i & 0 & 0 & G_i & 0 \\ 0 & 0 & -P_i & -L_i^\top & 0 & 0 \\ 0 & 0 & -L_i & 0 & 0 & L_i \\ 0 & G_i^\top & 0 & 0 & -H_i & 0 \\ 0 & 0 & 0 & L_i^\top & 0 & P_i \end{bmatrix} - \alpha_i \begin{bmatrix} [\bullet]\Phi_i \begin{bmatrix} I & X_i^+ \\ & Y_i \\ & -X_i \\ 0 & -U_i \\ & -V_i \end{bmatrix}^\top & 0 \\ 0 & 0 \end{bmatrix} \geq 0. \quad (1)$$

Gain computed by $K_i = L_i P_i^{-1}$.

Theorem 2 (Global stability condition) Assume that the i-th system is dissipative w.r.t. (F_i, G_i, H_i). Define

$$E_i := \begin{bmatrix} 0 & \cdots & I & \cdots & 0 \end{bmatrix}, \Lambda_i := [\bullet]^\top \begin{bmatrix} H_i - \beta_i I & G_i^\top \\ G_i & F_i \end{bmatrix} \begin{bmatrix} E_i & 0 \\ 0 & I \end{bmatrix},$$

$$\Theta_i := [\bullet]\ \Psi_i \begin{bmatrix} I & \tilde{V}_i \\ 0 & -\tilde{Y} \end{bmatrix}^\top, \hat{\Theta}_i := \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \Theta_i^{-1} \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}.$$

The global system is asymptotically stable, if $\exists \beta_i > 0$ and $\tau_i \geq 0$ s.t.

$$\Lambda_i - \tau_i \hat{\Theta}_i \geq 0. \quad (2)$$

Control algorithm

Key idea: (1) and (2) are both an LMI + (F_i, G_i, H_i) appear linearly → treat (F_i, G_i, H_i) as decision variables

Algorithm 1 Data-driven decentralized control

1: **for** $i \in \{1, \ldots, k\}$ **do**
2: **Input:** Data $\{X_i, X_i^+, U_i, V_i, Y_i\}$, $\{\tilde{V}_i, \tilde{Y}\}$
3: Solve (1), (2) w.r.t. $P_i > 0,\ L_i, \alpha_i \geq 0,\ \beta_i > 0,\ \tau_i \geq 0,\ (F_i, G_i, H_i)$
4: Compute $K_i = L_i P_i^{-1}$
5: **end for**
6: **return** $\{K_i\}_{i=1}^k$

- Control design & implementation: data-driven & decentralized
- Issues: complexity scales with the number of subsystems & each subsystem needs to know its "ordering"

Diffusive coupling case

Diffusive coupling: interconnection relation given by

$$v_i = \sum_{j \in \mathcal{N}_i} a_{ij}(y_j - y_i) \quad \forall i,$$

where $\mathcal{N}_i$: neighbors, a_{ij}: unknown

- Weighted degree: $d_i := \sum_{j \in \mathcal{N}_i} a_{ij}$
- Martinelli *et al.* [1]: if i-th system is dissipative w.r.t. (F_i, G_i, H_i) with

$$G_i = \frac{1}{2}\alpha I,\ -\frac{1}{2d_i} I < F_i < 0,\ H_i > 2 d_i \tilde{\alpha} I, \quad (3)$$

then the global system is asymp. stable

Idea: compute the upper bound of d_i from data and use it with (3)

Theorem 3 (Upper bound of d_i) The largest d_i consistent with data:

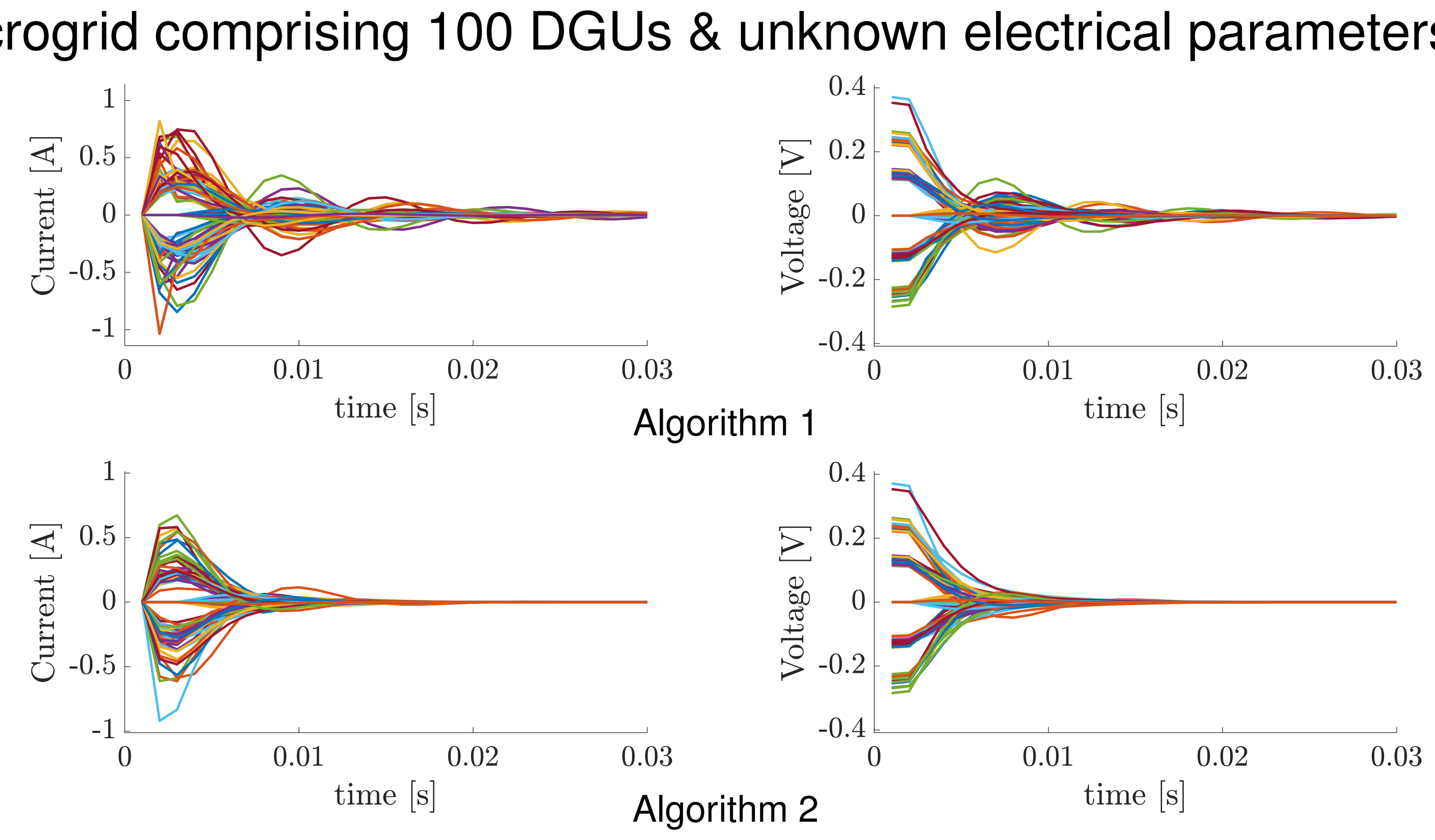
$$d_i^{\max} = \max_{d_i} d_i \text{ such that } \begin{bmatrix} \bullet \\ \bullet \end{bmatrix}^\top [\bullet]^\top \hat{\Theta}_i \begin{bmatrix} -E_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ d_i \end{bmatrix} \geq 0. \quad (4)$$

Algorithm 2 Data-driven decentralized control for diffusive coupling

1: **for** $i \in \{1, \ldots, k\}$ **do**
2: **Input:** Data $\{X_i, X_i^+, U_i, V_i, Y_i\}$, $\{\tilde{V}_i, \tilde{Y}\}$
3: Compute $d_i^{\max}$ from (4) and $d_i \leftarrow d_i^{\max}$
4: Solve (1), (3) w.r.t. $P_i > 0,\ L_i, \alpha_i \geq 0,\ (F_i, G_i, H_i)$
5: Compute $K_i = L_i P_i^{-1}$
6: **end for**
7: **return** $\{K_i\}_{i=1}^k$

Interconnected microgrid example

Microgrid comprising 100 DGUs & unknown electrical parameters:



[1] Andrea Martinelli, Ahmed Aboudonia, and John Lygeros. Interconnection of (Q, S, R)-Dissipative Systems in Discrete Time. *arXiv preprint, 2024.*

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